U. Clarenz, U. Diewald, M. Rumpf, A. Telea
Duisburg University

Numerical Analysis and Scientific Computing
[clarenz, diewald, rumpf]@math.uni-duisburg.de

Robust Classification of Surfaces

1 Introduction

The stable local classification of discrete surfaces with respect to features such as edges and corners or concave and convex regions respectively is as well difficult as indispensable for many surface processing applications. Usually, the feature detection is done via a local curvature analysis. In our work on classification, we want to use an additional local classification method on surfaces which avoids the evaluation of discretized curvature quantities. This method provides an indicator for smoothness of a given discrete surface and comes together with a built-in multi scale. The proposed classification tool is based on local zero and first moments on the discrete surface. The corresponding integral quantities are stable to compute and they give less noisy results compared to discrete curvature quantities. The stencil width for the integration of the moments turns out to be the scale parameter.

Typical surface processing applications are the segmentation on surfaces, surface comparison and matching and surface modelling.

2 Curvature analysis

In this section, we will present an overview on the classification based on curvature. The quantity for the detection of highly curved surface areas - namely edges - is in codimension 1 represented by the symmetric shape operator $S_{\mathcal{T}_x\mathcal{M}}$. An edge is supposed to be indicated by one sufficiently large eigenvalue of $S_{\mathcal{T}_x\mathcal{M}}$.

For the computation of the shape operator on discrete surfaces we use local L^2 projection of the surface onto quadratic polynomial graphs.

For stability reasons, we compute a shape operator $S_{\mathcal{T}_x\mathcal{M}_\epsilon}$ on $\mathcal{M}_\epsilon$. Here the parameter ϵ either indicates a "geometric Gaussian" filter-width, or the size of the neighbourhood we take into account for the local L^2 projection [4].

Now, we introduce for every point on $\mathcal{M}_\epsilon$, a classification tensor $a^\epsilon_{\mathcal{T}_x\mathcal{M}_\epsilon}$. It is supposed to be a symmetric, positive definite, linear mapping on the tangent space $\mathcal{T}_x\mathcal{M}_\epsilon$. Let us suppose that $w^{1,\epsilon}$, $w^{2,\epsilon}$ are the principal directions of curvature - the orthogonal eigendirections of the shape operator - and $\kappa^{1,\epsilon}$, $\kappa^{2,\epsilon}$ the principle curvatures - the corresponding eigenvalues. Then we define the tensor $a^\epsilon_{\mathcal{T}_x\mathcal{M}_\epsilon}$ in the basis $\{w^{1,\epsilon}, w^{2,\epsilon}\}$ as follows:

$$a^\epsilon_{\mathcal{T}_x\mathcal{M}_\epsilon} = \begin{pmatrix} G\left(\kappa^{1,\epsilon}\right) & 0 \\ 0 & G\left(\kappa^{2,\epsilon}\right) \end{pmatrix}, \qquad (1)$$

where the function G is given by $G(x) := \frac{1}{1+(x/\beta)^2}$. Here β serves as a user defined threshold parameter which classifies the significance of surface features. Hence, a point is supposed to belong to an edge if there is one principal direction of curvature on $\mathcal{M}_\epsilon$ with large curvature compared to β. If the second principal curvature is small w.r.t. β, we consider the first direction as being orthogonal to an edge on the surface. At corners both principal curvatures of $\mathcal{M}_\epsilon$ are large. Summarizing this, our tensor leads to the following surface classification:

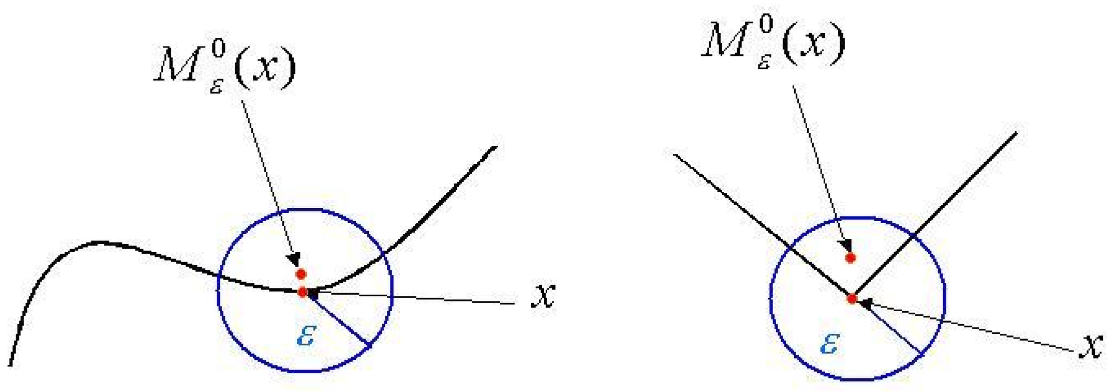
- *Smooth areas* are characterized by $a^\epsilon_{\mathcal{T}_x\mathcal{M}_\epsilon} \backsim \mathrm{diag}[1,1]$.
- *Edges* are defined by $a^\epsilon_{\mathcal{T}_x\mathcal{M}_\epsilon} \backsim \mathrm{diag}[1,0]$. In this case, the edge direction is given by $w^{2,\epsilon}$ and we assume $|\kappa^{1,\epsilon}| >> |\kappa^{2,\epsilon}|$.
- *Corners* are defined by $a^\epsilon_{\mathcal{T}_x\mathcal{M}_\epsilon} \backsim \mathrm{diag}[0,0]$.

3 Classification via moments

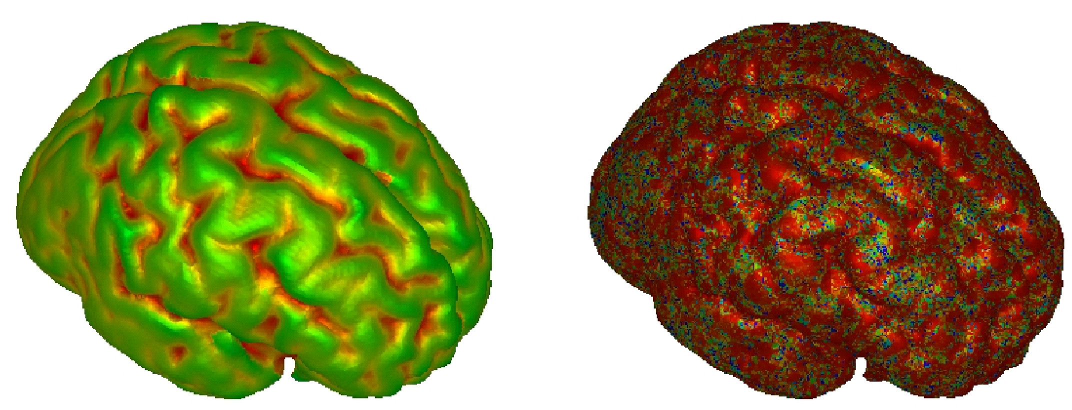
Let us consider first the zero moment classification: We compute for $x : \mathcal{M} \to \mathbb{R}^{d+1}$, $d = 1, 2$, the barycenter $M^0_\epsilon(x(\xi)), \xi \in \mathcal{M}$, of $x(\mathcal{M}) \cap B_\epsilon(x(\xi))$, where $B_\epsilon(x(\xi))$ is the Euclidian ϵ-ball. In smooth domains of a surface, the difference $M^0_\epsilon(x(\xi)) - x(\xi)$ scales quadratically in ϵ. We have the relation:

$$M^0_\epsilon(x(\xi)) - x(\xi) = -\epsilon^2\, c(d)\; hn \,+ o(\epsilon^2),$$

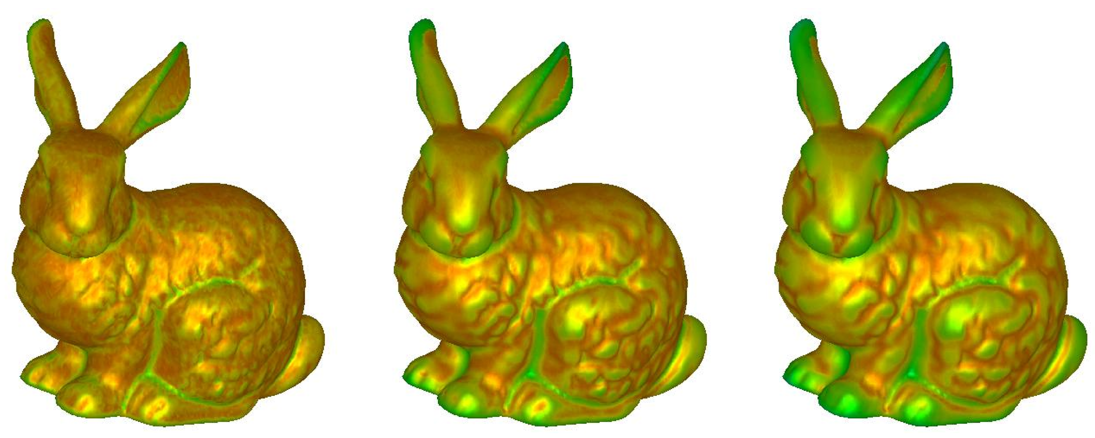
where $c(d)$ is a constant depending only on the surface dimension and hn is the mean curvature vector. In a non-smooth situation we have only linear scaling of the zero moment shift. Thus $|M^0_\epsilon(x(\xi)) - x(\xi)|/\epsilon$ may serve as an edge indicator.



The above figure shows a smooth situation on the left hand side and an edge-situation on the right. The different scaling of the zero moment shift indicates smoothness resp. features.



Here, we compare a moment based classification of a human cortex data-set and the corresponding curvature based classification. Obviously, using zero moments leads to higher robustness. Below the detection of details on different scales choosing a different radius ϵ is shown:



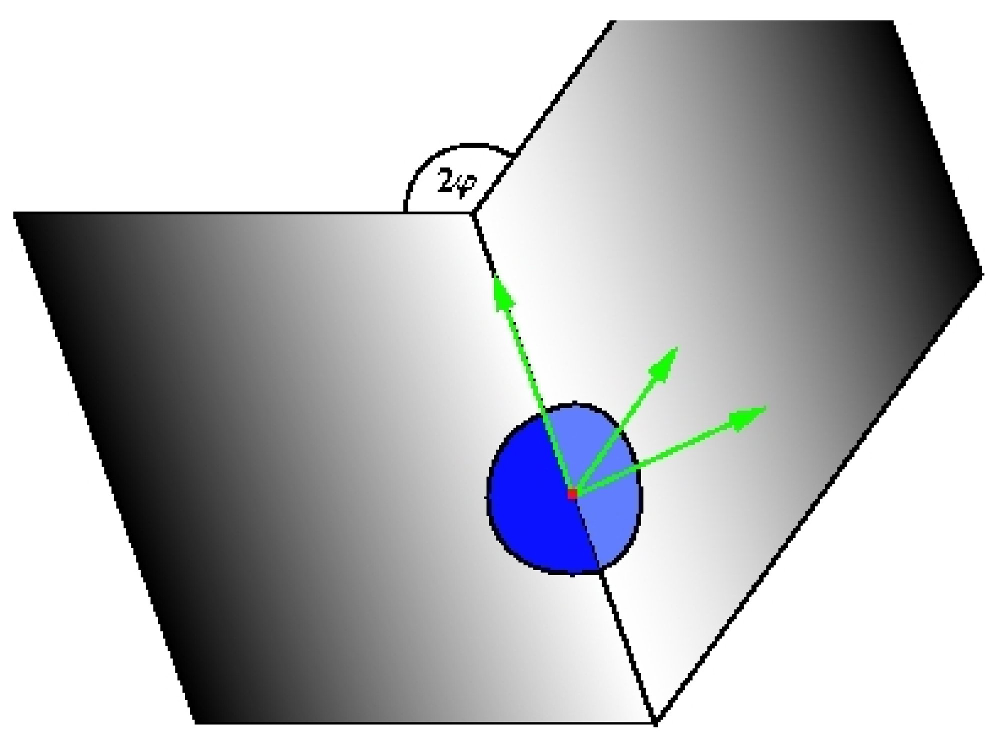
Let us introduce the first order moment now. It is defined as

$$M^1_\epsilon(x(\xi)) = \fint_{B_\epsilon\cap\mathcal{M}} (x - M^0_\epsilon(x(\xi))) \otimes (x - M^0_\epsilon(x(\xi)))\, dA\,,$$

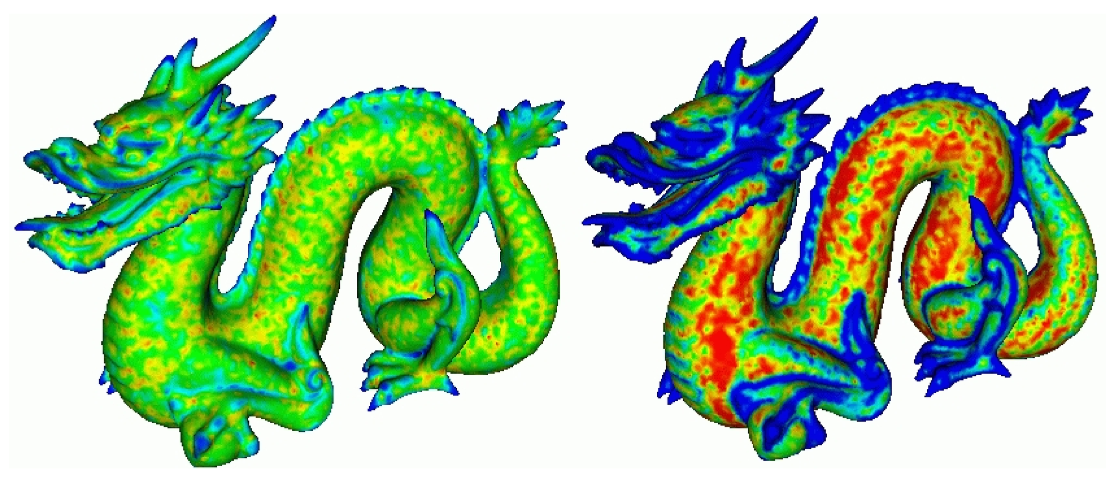
where $y \otimes z := (y_i z_j)_{i,j=1,\ldots,d+1}$ and the meaning of ϵ is as in the zero moment case the radius of a Euclidean ball. The scaling of the first moment is in smooth and in non-smooth cases quadratic, i.e.,

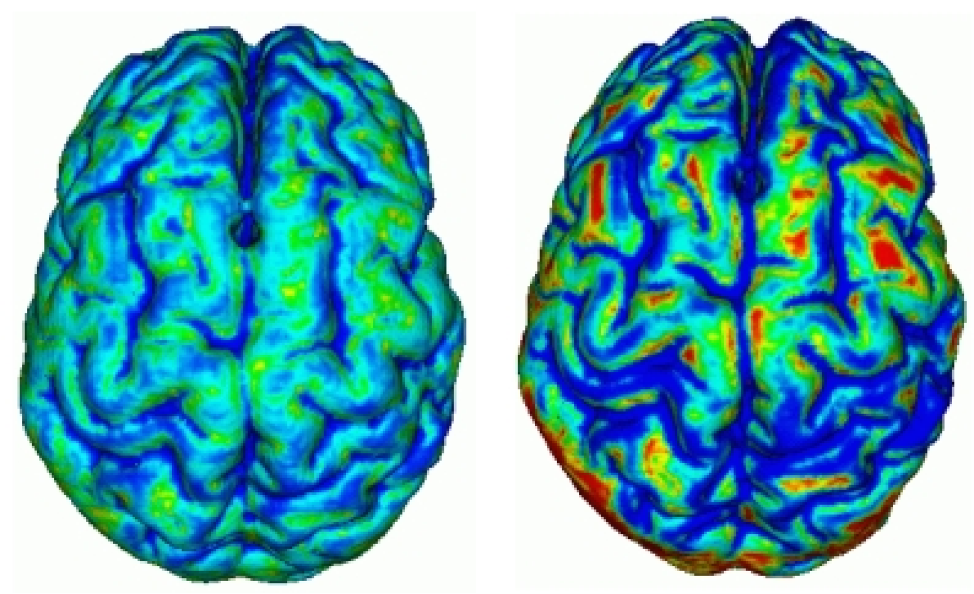
$$M^1_\epsilon(x(\xi)) = \epsilon^2\; 2c(d)\;\Omega \,+ o(\epsilon^2).$$

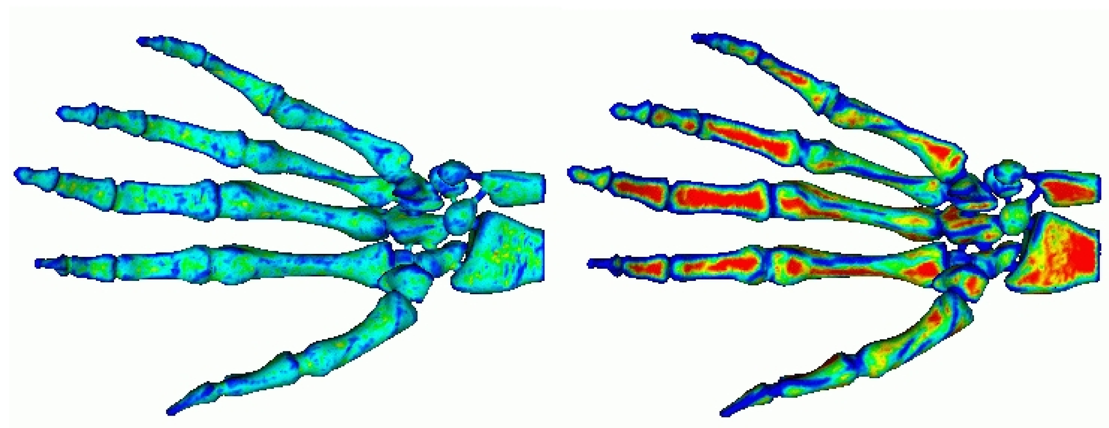
In this case we are able to distinguish between smooth areas and features by an eigenvalue analysis of the matrix Ω. In smooth cases, Ω is the projection onto the corresponding tangent space, where in the non-smooth case, all eigenvalues are different from 0.



The above figure shows typical eigenvectors of the first moment in an edge situation. This information enables us to determine the direction of an edge. Furthermore, the incorporation of first moment information leads to even more robust results. This will become clear by the following images. On the left hand side, the classification result just using zero moments is shown. The radius ϵ is chosen relatively small. On the right hand side, we see a classification, where also first moments are taken into account.







Finally, let us mention that we can use of these classification results as basis of a diffusion tensor for surface fairing. Generalizing the algorithm introduced by Dziuk [3] for mean curvature flow, we can define an anisotropic geometric diffusion process, which smoothes the surface in regions being classified as smooth and preserves edges on the surface. In addition we can allow smoothing along an edge. The corresponding generalized mean curvature flow is given by the following equation:

$$\partial_t x - [\mathrm{div}_\mathcal{M}(a^\epsilon_{\mathcal{T}_x\mathcal{M}} \nabla_\mathcal{M} x) \cdot n]\, n = 0\,,$$

where n denotes the surface normal. The tensor $a^\epsilon_{\mathcal{T}_x\mathcal{M}}$ is similarly chosen as in the curvature based analysis. In the direction of an edge, diffusion is allowed, where across an edge, we reduce diffusion. For a more detailed discussion of this type of equation, we refer to [1]. Below, different time-steps of this diffusion process starting with a noisy initial configuration are shown.

References

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[4] T. Preusser and M. Rumpf, *A level set method for anisotropic geometric diffusion in 3D image processing*, To appear in SIAM J. Appl., (2002).