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Anisotropic Geometric Diffusion in 3D Image Analysis

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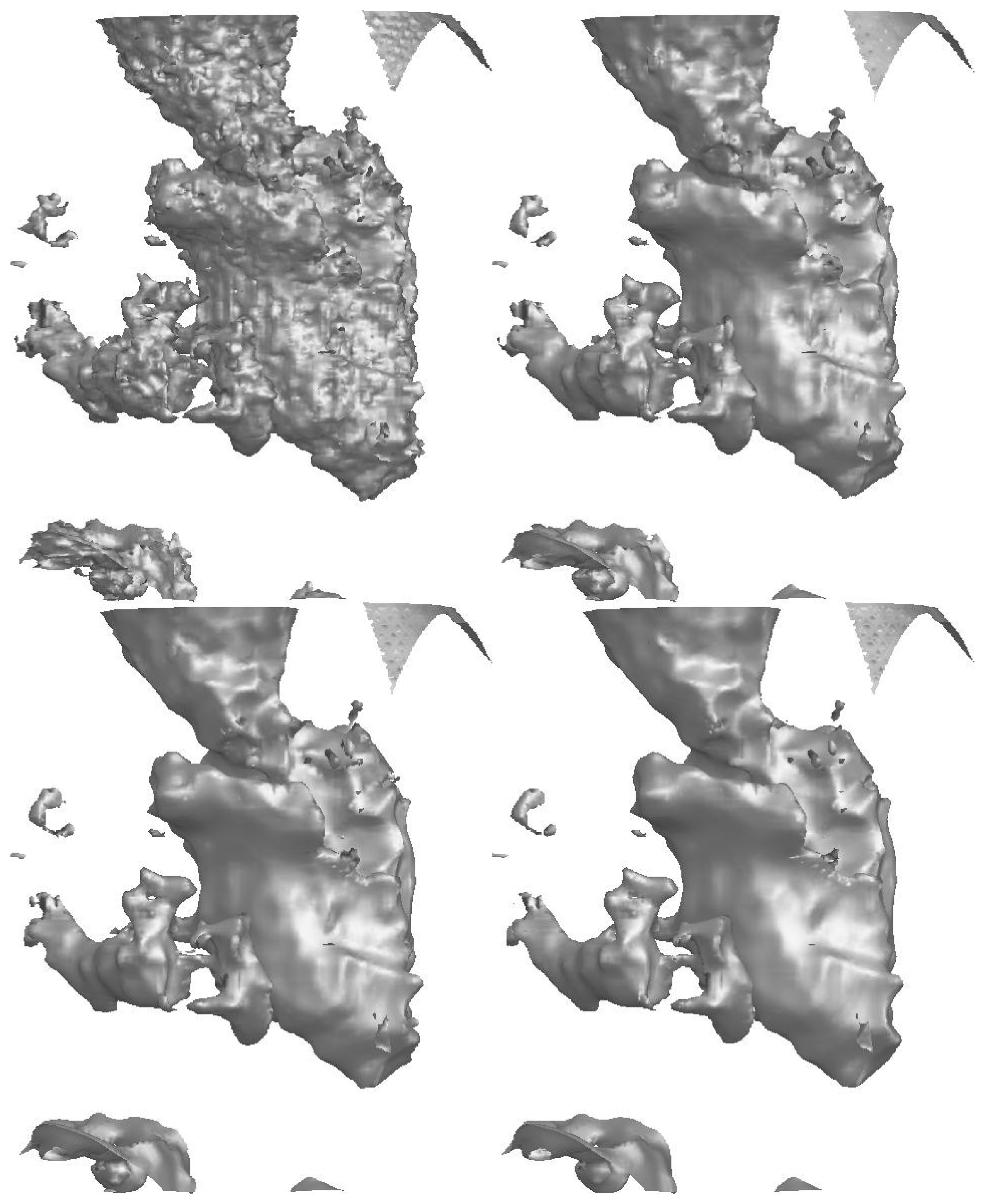
1 Introduction

We discuss a new anisotropic level set method for the denoising of large, digital 3D images (cf. [2] for a related method). The method is able to preserve edges and corners on level sets while still allowing tangential smoothing along the edges and it is characterized by a rich class of invariant shapes. The core of the method is an evolution driven by *anisotropic geometric diffusion* of level surfaces. Two parameters steer the performance of the method:

- A threshold value λ related to principal curvatures which indicate an edge and
- a filter width σ which controls the noise reduction on the surface before evaluating the shape operator.

The difference between the actual and a regularized shape operator plays an essential role in the control of the evolution problem.



2 Anisotropic Geometric Diffusion on Level Sets

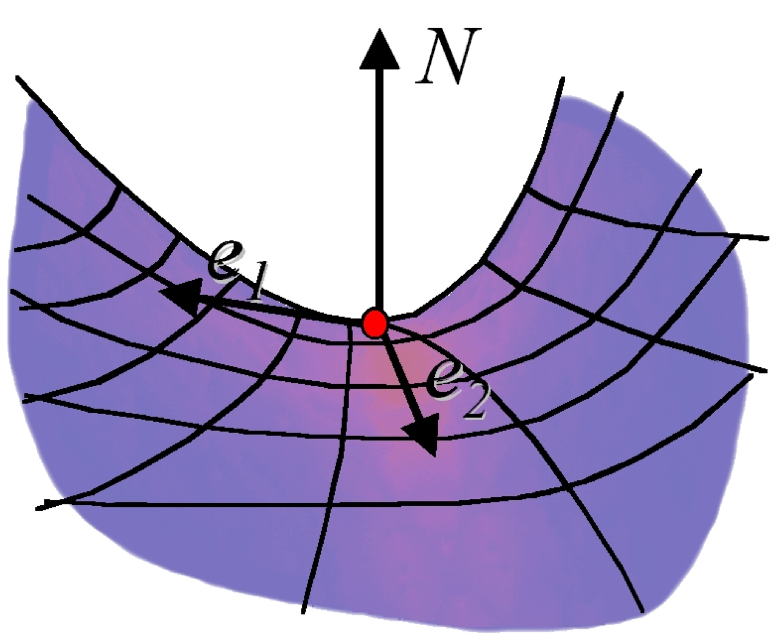
Given an initial 3D image ϕ_0 on a domain Ω, we ask for a scale of images $\{\phi(t,\cdot)\}_{t\geq 0}$ which obey the anisotropic geometric evolution equation:

$$\partial_t\phi - \|\nabla\phi\| \,\mathrm{div}\left(a^\sigma \frac{\nabla\phi}{\|\nabla\phi\|}\right) = 0 \qquad \text{on } \mathbb{R}^+\times\Omega,$$

$$a^\sigma \frac{\partial\phi}{\partial\nu} = 0 \qquad \text{on } \mathbb{R}^+\times\partial\Omega,$$

$$\phi(0,\cdot) = \phi_0(\cdot) \qquad \text{in } \Omega.$$

The diffusion tensor a^σ is supposed to care about the preservation of edges and the tangential smoothing along edges.



On a level set of the image the Jacobian of the normal $N := \frac{\nabla\phi}{\|\nabla\phi\|}$ (which coincides with the Shape Operator S on the tangent space) is characterized by its

- Eigenvalues = Principal Curvatures κ^1, κ^2
- Eigenvectors = Principal Directions of Curvature v^1, v^2

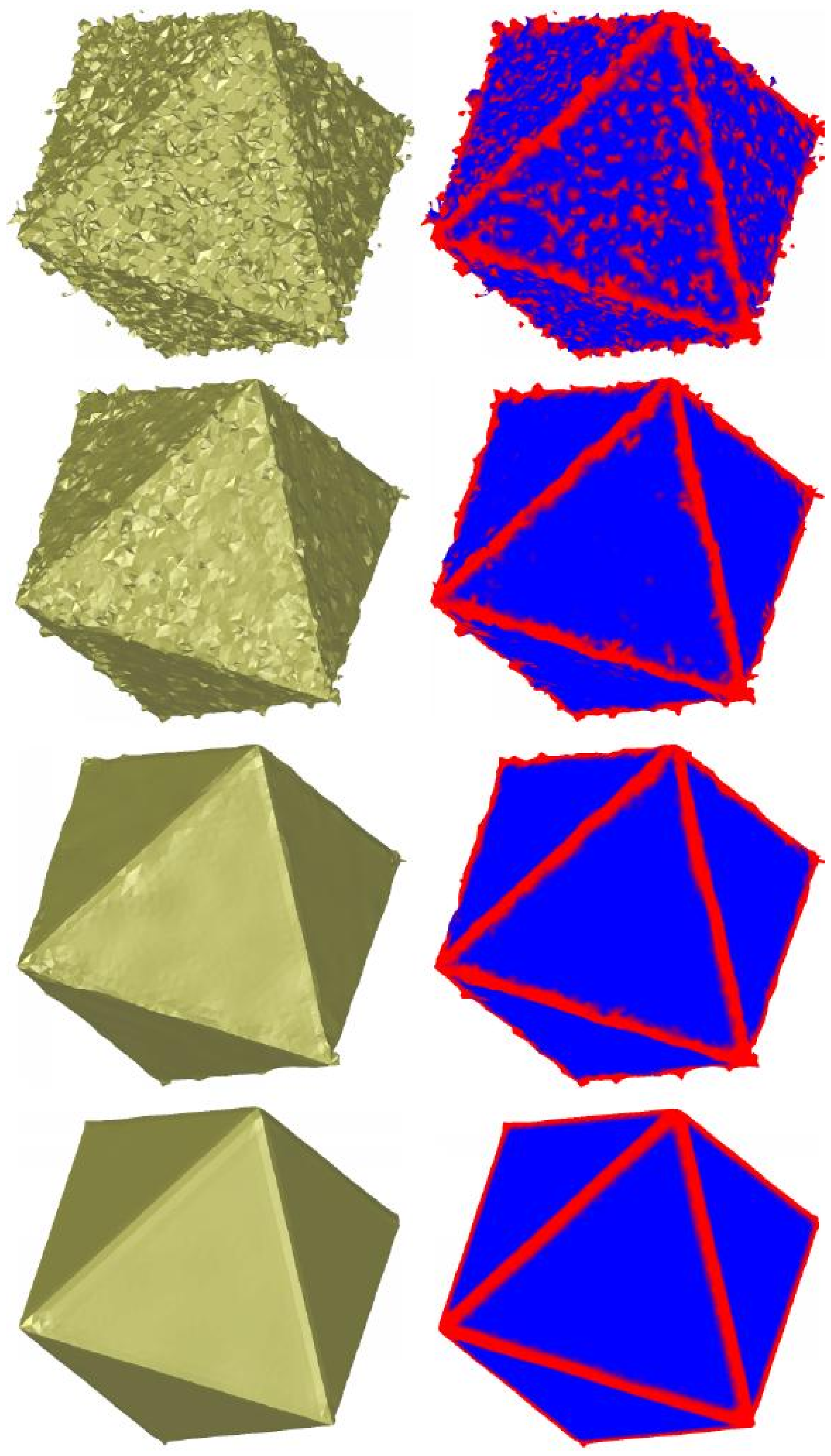
Now we define the anisotropic diffusion tensor a^σ as a function of a regularized shape operator S^σ

$$a^\sigma(S^\sigma) = \begin{pmatrix} G(\kappa^{1,\sigma}) & & \\ & G(\kappa^{2,\sigma}) & \\ & & 0 \end{pmatrix} \qquad G(s) = \frac{1}{1+s^2/\lambda^2}.$$

in a basis of $\{v^{1,\sigma}, v^{2,\sigma}, N^\sigma\}$. Furthermore, the problem can be rewritten as

$$\partial_t\phi + \|\nabla\phi\| \left(\mathrm{tr}(a^\sigma(S_\sigma - S)) + \mathrm{div}\; a^\sigma(N^\sigma - N)\right) = 0\,.$$

The evolution is mainly driven by the difference of a regularized and the true shape operator weighted by the anisotropic weights given from the diffusion tensor.



3 Regularized Shape Operator on Discrete Data

We have different choices for the definition of the regularization we need:

1. Short time evolution under mean curvature motion
$$\partial_t\phi - \|\nabla\phi\| \,\mathrm{div}\left(\frac{\nabla\phi}{\|\nabla\phi\|}\right) = 0.$$
2. Convolution of the image with a kernel K_σ of width σ, i.e. $\phi^\sigma := K_\sigma * \phi$

Definition of 2nd derivatives on trilinear images?

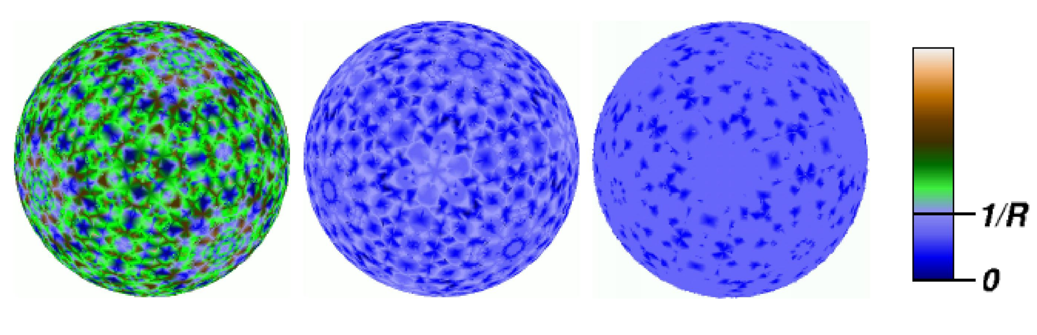
3. Local Regularization: Least squares fit (*local L^2 projection*) of the image ϕ onto a subspace $\mathcal{Q}$ of the space of quadratic polynomials $\mathcal{P}_2$, i.e.
$$\int_{\mathcal{B}_\sigma(x)} (\phi(y) - (\Pi_{x,\sigma}\phi)(y))\; q\; dy = 0 \qquad \forall q \in \mathcal{Q}.$$

We then compute curvature based on the Shape Operator S^σ of the projection $\Pi_{x,\sigma}\phi$.

Ellipsoidal levelsets remain invariant under the evolution, because then $\Pi_{x,\sigma}\phi = \phi \Rightarrow S^\sigma = S$.

4 Consistency of the Regularization

We evaluate the curvature on the projection of spherical level sets and compare the result with the true curvature.



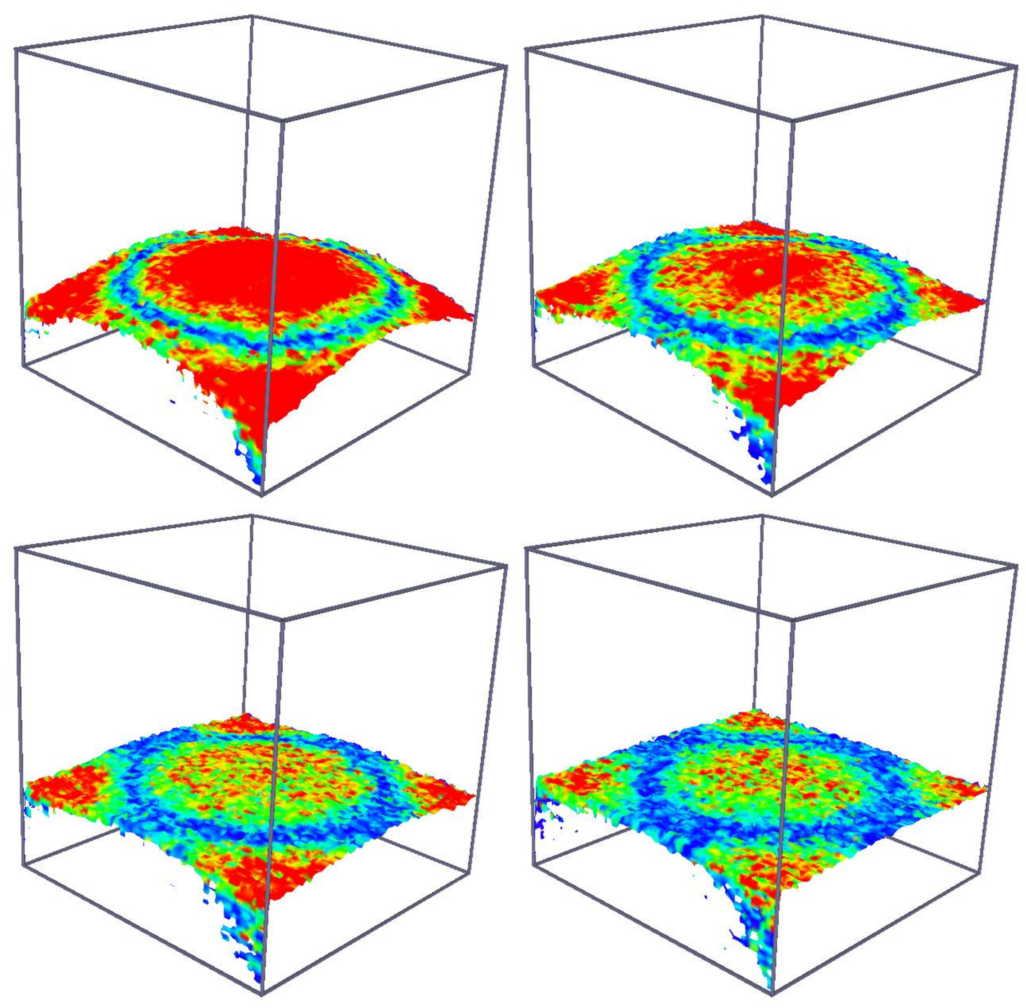
Therefore we define as an error $e := \left\|\kappa_i^\sigma - \frac{1}{R}\right\|_{L^2(\Omega_\sigma)}$.

l	$h=1/32$	$h=1/64$	$h=1/128$
2	5.451e-02	5.479e-02	5.493e-02
4	1.398e-02	1.405e-02	1.409e-02
8	3.517e-03	3.535e-03	3.544e-03
16	8.806e-04	8.852e-04	8.875e-04

5 Analysis of Image Sequences

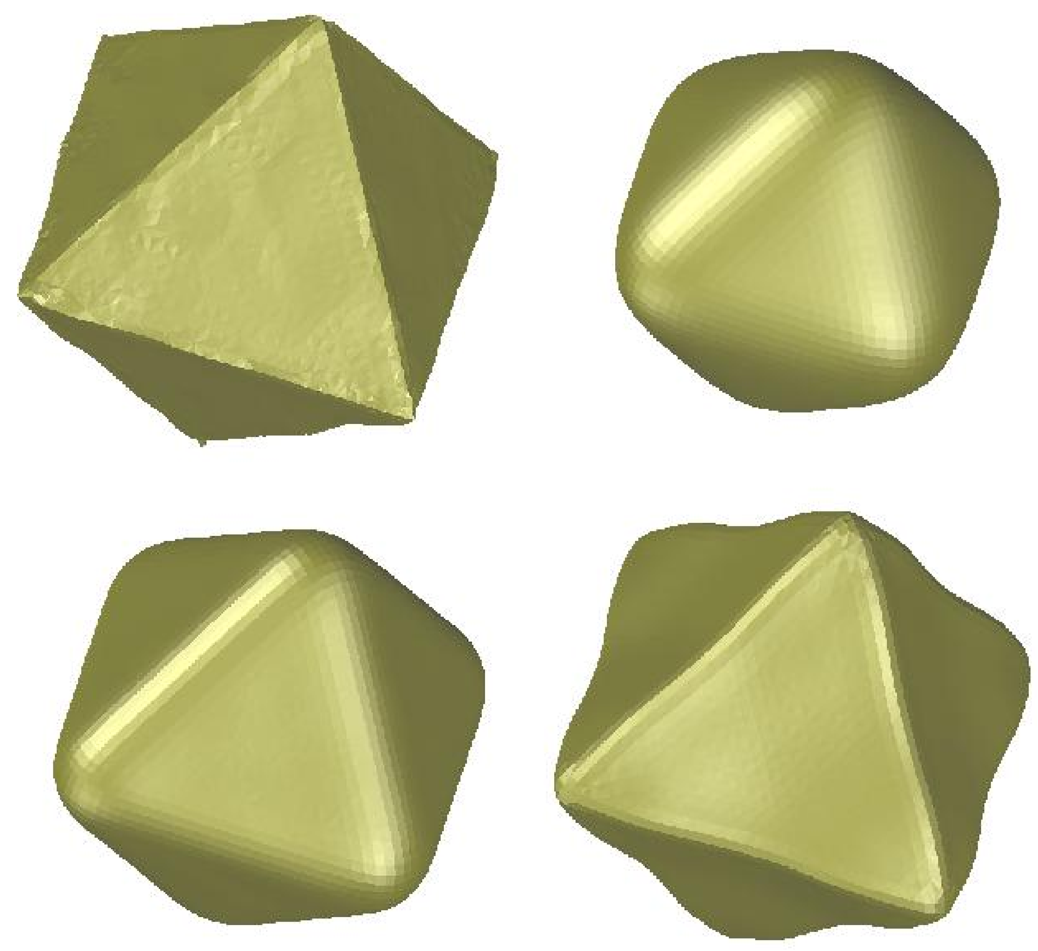
If we additionally consider sequences of image data where the level sets move in time we can

- extract velocity information from experimental data and
- apply a coupled space-time anisotropic diffusion filter.



6 Conclusions

The level set method for generalized geometric diffusion in 3D image processing invokes mainly geometric quantities of level sets of an image intensity. The morphological scale space method is able to successively smooth noisy initial data while retaining edges and corners of the level sets, indicated by considerably large principal curvatures.



The evaluation of the Shape Operator, the surface normal and the tangent space is based on a local L^2 projection onto a polynomial space. Therefore under the evolution ellipsoidal level sets remain invariant.

7 Bibliography

[1] T. Preusser and M. Rumpf. *A level set method for anisotropic geometric diffusion in 3D image processing*. SIAM J. Appl. Math., to appear, 2002.

[2] U. Clarenz, U. Diewald, and M. Rumpf. *Nonlinear anisotropic diffusion in surface processing*. In T. Ertl, B. Hamann, and A. Varshney, editors, Proceedings of IEEE Visualization 2000, pages 397-405, 2000.